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A REDUCED MODEL OF KINETIC EFFECTS RELATED TO THE SATURATION OF STIMULATED BRILLOUIN SCATTERING

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We developed a reduced description of kinetic effects that is included in a fluid model of stimulated Brillouin backscattering (SBS) in low Z plasmas (e.g. He, Be). Following hybrid-PIC simulations,¹ the modified ion distribution function is parametrized by the width δ of the plateau created by trapping around the phase velocity of the SBS-driven acoustic wave. An evolution equation is derived for δ , which affects SBS through a frequency shift and a reduced Landau damping. This model recovers the linear Landau damping value for small waves and the time-asymptotic nonlinear frequency shift calculated by Morales and O'Neil.³ Finally we compare our reduced model with Bzohar⁴ simulations of a Be plasma representative of experiments that have shown evidence of ion trapping.⁵

I. INTRODUCTION

We are trying to model the effect of ion trapping on stimulated Brillouin scattering (SBS) in a fluid code (pF3d⁶). From Bzohar⁴ hybrid particle-in-cell (PIC) simulations, we have determined that the main effects of ion trapping are a nonlinear frequency shift and a reduction of the Landau damping from its initial linear value^{1,2}, due to the modifications of the ion distribution function around the phase velocity v_ϕ of the SBS-driven acoustic waves. In absence of relaxation processes (collisions, diffusion, advection), the nonlinear frequency shift and reduced damping persists even when no large-amplitude wave is present in the plasma. Initial bursts of SBS, associated with large amplitude driven acoustic waves, modify the background distribution function and thus the response of the plasma to subsequent stimulation. Fig. 1 shows the evolution of the SBS reflectivity R_{sbs} as observed in a Bzohar simulation. The plasma parameters are representative of experiments done at TRIDENT⁵: a 500 μm Be plasma with $T_e = 430$ eV, $T_i = T_e/2$ and $n_e = 10^{20} \text{ cm}^{-3}$. The intensity of the interaction beam was $1.25 \times 10^{14} \text{ W cm}^{-2}$ with a wavelength $\lambda_0 = 0.53 \mu\text{m}$. These parameters are used for numerical examples throughout this paper, if not otherwise precised. Fig. 2 shows the modified ion distribution function at $t = 250\text{ps}$, and the resulting frequency shift that detunes SBS at later time. More details can be found in reference 3. In section II, we derive our model describing the formation of a plateau around v_ϕ in the ion distribution function $f(v)$ and its implications in term of resonant acoustic frequency and Landau damping. In section III, we introduce various loss terms that tend to relax $f(v)$ towards a Maxwellian. Section IV describes the implementation of the model in the fluid code pF3d. Finally, in section V we compare PIC simulations with fluid simulations including the reduced model described in this paper.

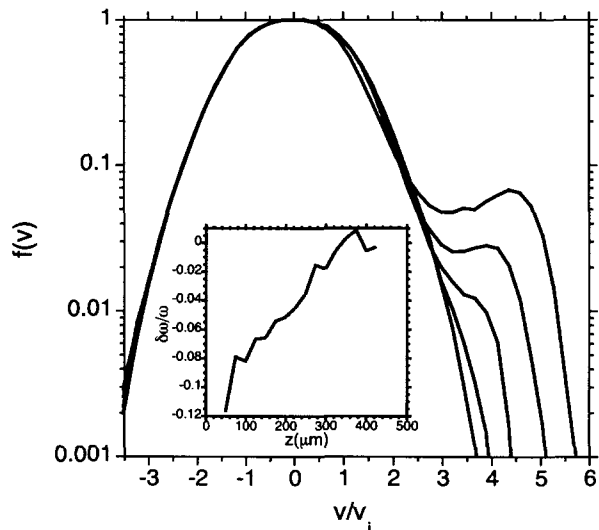


FIG. 1: Plot of the ion distribution function averaged over $25\mu\text{m}$ every $100\mu\text{m}$ in the plasma, at $t = 250$ ps (from left to right, $z=50, 150, 250, 350, 450 \mu\text{m}$). The tail results in a spatial variation of the local resonant acoustic frequency corresponding to the SBS driven wave ($k_a = 2k_0$), as shown in the insert.

II. NONLINEAR FREQUENCY SHIFT AND REDUCED DAMPING

Our goal is to model the effects of trapping-induced modifications of $f(v)$ on the dispersion of acoustic waves. We parametrize the (locally averaged) ion distribution function as:⁷

$$f(v) = f_0(v) + \beta f_1(v - v_\phi) + \gamma f_2(v - v_\phi), \quad (1)$$

$$f_0(v) = \exp(-v^2/2v_i^2)/[(2\pi)^{1/2}v_i], \quad (2)$$

$$f_1(x) = x \exp(-x^2/2\delta^2), \quad (3)$$

$$f_2(x) = (\delta^2/3)(x^2/\delta^2 - 1) \exp(-x^2/2\delta^2); \quad (4)$$

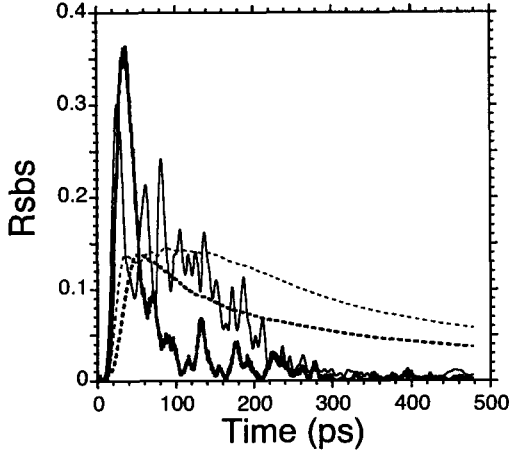


FIG. 2: Time history of R_{sbs} as observed in a PIC simulation (Bzohar, thick lines) and in a fluid model including our reduced model (thin lines). The solid lines correspond to the instant reflectivity, while the dashed lines show the time-averaged reflectivity.

f_1 and f_2 conserve by construction the number of particles. The effect of trapping on the locally-averaged distribution function is a flattening of $f(v)$ around $v = v_\phi$, thus one can reasonably choose $\beta = -\partial f_0 / \partial v|_{v=v_\phi}$ and $\gamma = -\partial^2 f_0 / \partial v^2|_{v=v_\phi}$. The only parameter left is δ , the width in velocity space of the modifications. Our model will consist in an equation for the evolution of $\delta(x, t)$, coupled with a description of the linear response of the plasma assuming a distribution function $f(v, \delta(x, t))$. This approach is justified by a careful analysis of SBS-PIC simulations, showing that the frequency of the driven acoustic wave tends to follow the solution of the linear dispersion relation calculated with the locally averaged (over a few wavelengths) distribution function.¹

In presence of a finite-amplitude acoustic wave (amplitude $\delta n/n$ and wavelength $2\pi/k$), PIC-simulations and analytical calculations have shown that the flattening of $f_0(v)$ occurs over $[v_\phi - v_{tr}, v_\phi + v_{tr}]$, where $v_{tr} = \sqrt{Z T_e / M_i} \sqrt{|\delta n/n|}$ is the trapping velocity and a steady-state is reached in a few ω_b^{-1} , where $\omega_b = k v_{tr}$ is the bouncing frequency of trapped ions. In order to be more quantitative, we now calculate the rate of increase in kinetic energy due to a variation of δ and try to recover the linear Landau damping result when $\delta n/n \ll 1$. One finds:

$$\begin{aligned} E_1 &= \frac{M_i n_i}{2} \int v^2 f_1(v - v_\phi) dv = -\sqrt{(2\pi)} M_i n_i \beta v_\phi \delta^2 / 5 \\ E_2 &= \sqrt{2\pi} M_i n_i \gamma \delta^3 / 3. \end{aligned} \quad (6)$$

$E_2/E_1 \simeq (Z T_e / 3 T_i) \delta n/n \ll 1$, when kinetic effects are important f_1 accounts for energy transfer. Comparing the rate of increase in kinetic energy dE_1/dt with the

energy of the local acoustic wave $W_{ac} = \frac{\partial(\omega\epsilon)}{\partial\omega}|_{\omega=kv_\phi} \frac{|E_a|^2}{16\pi}$ gives an effective damping $\nu_i = dE_1/2W_{ac}dt$:

$$\nu_i = \nu_{Li} \frac{6\sqrt{2\pi}\delta^2 d\delta}{v_{tr}^3 \omega_b dt} \quad (7)$$

where

$$\nu_{Li} = \frac{-\beta \omega_{pi}^2 / k^2}{\partial\epsilon/\partial\omega|_{\omega=kv_\phi}}$$

is the usual ion contribution to the linear Landau damping. One is thus led to choose:

$$\frac{d\delta^3}{dt} = \frac{\omega_b}{2\sqrt{2\pi}} v_{tr}^3 \quad (8)$$

as an equation for the time evolution of δ when $\delta < v_{tr}$. This implies that $\nu_a = \nu_{Li} + \nu_{Le}$ where ν_{Le} is the electron contribution to the Landau damping and ν_a is the total damping affecting the SBS-driven acoustic wave. When $\delta(x, t) > v_{tr}(x, t)$, the distribution function has already been flattened and δ no longer grows. In that situation, the ion Landau damping is reduced to zero and $\nu_a = \nu_{Le}$ which represents a large reduction in the damping rate for low Z plasmas. At the same time, the modification of the distribution function leads to a frequency shift in the dispersion relation for acoustic waves. A perturbative analysis of the linear Vlasov dispersion relation using $f(v)$ gives:⁷

$$\delta\omega = \frac{2\sqrt{2\pi}}{3\partial\epsilon_0/\partial\omega} \frac{\omega_{pi}^2}{k^2} \gamma \delta \quad (9)$$

$$= -0.83 \omega v_\phi^2 \gamma \delta, \quad (10)$$

which agrees with the result of reference³ if one chooses $\delta = v_{tr}$. A similar agreement between a quasi-linear-like calculation and a detailed nonlinear calculation including effects of trapped particles can be found in reference⁸ for Langmuir waves. No simple explanation was found for such a close agreement.

III. LOSS TERMS

Various processes tend to restore a Maxwellian distribution function. Here we address the effects of diffusion in space, advection by an external flow and collisions.

In a RPP-smoothed beam, the SBS-driven acoustic waves tend to grow to larger amplitudes in laser speckles, where the laser intensity is higher. These speckles have a narrow and elongated shape, and transverse advection of hot ions can restore a Maxwellian distribution function, as was suggested in reference⁸ for a single speckle experiment. In the case of a wide RPP-smoothed beam, numerous speckles coexist parallel to each other, and the hot ion tail will diffuse among them, leading to a global modification of the background distribution function. To

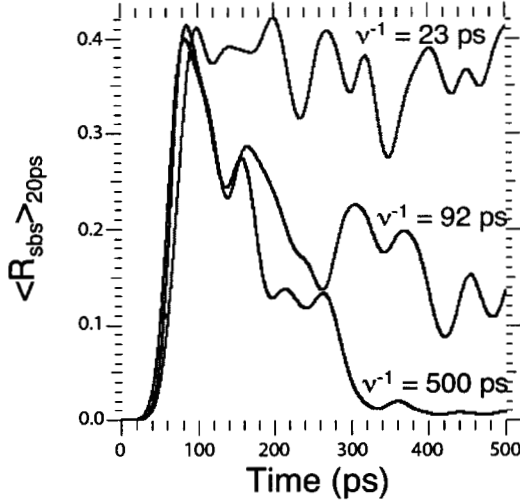


FIG. 3:

model this, we propose a transverse diffusion equation for δ :

$$(\partial_t - D\Delta_\perp)\delta = 0. \quad (11)$$

Dimensional analysis leads to $D \equiv L^2/T$; the typical spatial scale is the width of a speckle: $L = f^\# \lambda_0$, where $f^\#$ is the numeric aperture of the laser beam and λ_0 its wavelength. The timescale is the transit time of thermal ions across a speckle (remember that trapped ions are only accelerated along the longitudinal axis of the speckles) $T = f^\# \lambda_0 / v_i$, which gives $D = v_i f^\# \lambda_0$.

A transverse flow, often present in experiments due to the expansion of a heated plasma, advect the hot ions outside of the interaction beam, thus restoring a Maxwellian distribution function.⁸ Fig. ?? shows the SBS reflectivity for three different one-dimensional simulations in which a loss term accounts for transverse advection at Mach 1 with various interaction beam diameter L_\perp . This effective loss term is defined as $\nu = L_\perp / v_\phi$. $\nu^{-1} = 23\text{ps}$ corresponds to a diffraction limited beam with an $f/8$ aperture. The ion distribution function remains close to a Maxwellian and R_{sbs} is large during the entire simulation as there is no frequency shifts and detuning of the instability. $\nu^{-1} = 500\text{ps}$ corresponds to a smoothed beam with a $70 \mu\text{m}$ diameter. In this case, the transverse advection of hot ions is too slow to affect the reflectivity.

Ion-ion collisions can also relax $f(v)$ towards $f_0(v)$. Here we are interested in collisions of fast ions ($v \simeq v_\phi$) with thermal ions ($v \simeq v_i$).

A velocity plateau of half-width δ around $v = v_\phi$ dissipates through parallel diffusion as:

$$\frac{d\delta^2}{dt} = -\nu_\parallel v_\phi, \quad (12)$$

where $\nu_\parallel = \nu_0(v_i^2/v_\phi^2)$ and $\nu_0 = 4\pi Z^4 n_i e^4 \Lambda_{ii} / M_i^2 v_\phi^3$ is the relaxation rate for test ions moving at $v = v_\phi$ in a background of thermal ions $v = v_i$.⁹

IV. PF3D IMPLEMENTATION

For numerical efficiency, the evolution of the plateau width δ is split into an equation for the driving term and loss due to collisions (Eq.13), followed by the effect of advection and diffusion (Eq.15). In the latter have been included the effect of an external flow u on the hot ion tail and its advection at an average speed v_ϕ along the laser beam propagation axis.

$$\partial_t \delta^3 = H(\delta < \alpha v_{tr}) \frac{\omega_b}{2\sqrt{2\pi}} v_{tr}^3 - \frac{3}{2} \nu_0 v_i^2 \delta; \quad (13)$$

$$\delta = \text{Max}(0, (\delta^3)^{1/3}); \quad (14)$$

$$(\partial_t + u \cdot \nabla + v_\phi \partial_z - D\Delta_\perp)\delta = 0. \quad (15)$$

Knowing δ , one can compute the response of the plasma in term of a frequency shift for acoustic waves and an effective damping ν_a following:

$$\frac{\delta\omega}{\omega} = v_\phi^2 \gamma \delta; \quad (16)$$

$$\nu_a = \nu_{Le} + \nu_{coll} + \nu_i(\delta); \quad (17)$$

$$\nu_i(\delta) = \nu_{Li} \times H(\delta < \alpha v_{tr}). \quad (18)$$

Here we introduce a (small) additional damping ν_{coll} due to collisions. $\alpha = O(1)$ is a free parameter which comes from our arbitrary parametrization for $f(v)$ where there is no sharp cut-off in δ to define the exact width of the plateau in $f(v)$. The (nonlinear) damping ν_a and frequency shift $\delta\omega$ are then used in the equation describing the evolution of the SBS-driven acoustic wave, as described in reference¹⁰.

V. COMPARISON WITH BZOHAR SIMULATIONS

The model described in the previous section has been implemented in the fluid code pF3d and the results can be compared with Bzohar simulations. Here we use the laser-plasma parameters detailed in the introduction. In pF3d, a broadband volumic noise source models the thermal acoustic fluctuations and seeds SBS. In order to compare with the PIC simulations, where the noise source is due to the finite number of particles and usually much higher than thermal levels, the noise source was scaled until the Thomson scattering levels (i.e. SBS at very low intensity) given by the two codes were similar. Fig. ref1 shows a good qualitative agreement in the time history of R_{sbs} . As the detailed evolution (i.e. bursts) is sensitive to the exact realisation of the noise source, it is difficult to be more quantitative. Fig. 4 shows that the nonlinear frequency shifts calculated are similar, as well

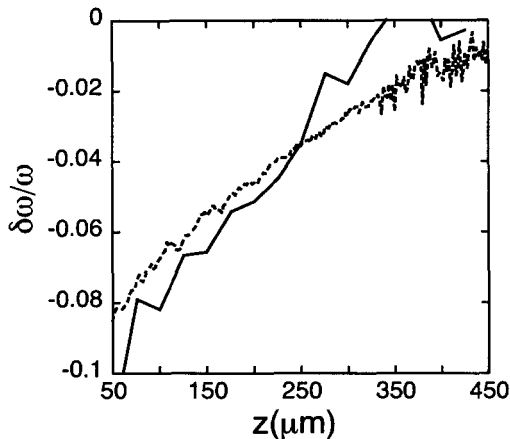


FIG. 4: Nonlinear detuning due to trapping effects, as observed in Bzohar simulations (solid line) and modeled in our fluid code (dashed line).

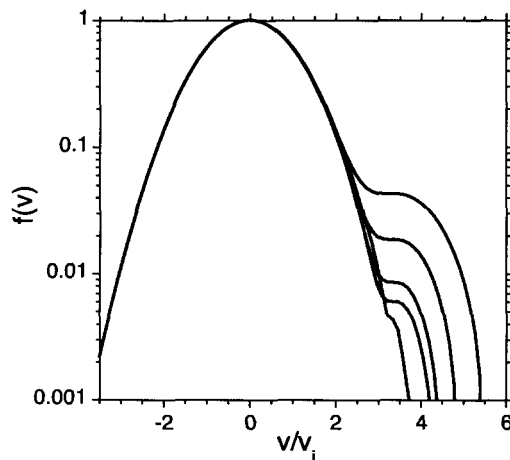


FIG. 5: Modified ion distribution functions at successive positions in the plasma (c.f. Fig. 2), as given by the parametrization shown in Section II. It compares well with Fig. 2.

as the corresponding modified ion distribution functions at different positions in the plasma. We found that choosing $\alpha = 2$ produced the best match between these simulations. No other free parameter was used. It is likely that α will have to be fine-tuned for different plasma parameters. A parametric study is underway.

VI. CONCLUSION

Hybrid-PIC simulations of SBS have shown that kinetic processes (trapping of ions) affect SBS through frequency shifts and modified dampings for the driven acoustic wave. We have presented a reduced model of this effects that has been implemented in a fluid model of SBS. This model is computationally efficient (one parameter, local in space) and by construction recovers the linear Landau damping for small acoustic waves as well as the asymptotic frequency shift calculated by Morales and O'Neil. Comparisons with 1D hybrid-PIC simulations are promising, but a systematic study and two-dimensional comparisons will be necessary before this model can be fully validated.

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